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Saskatchewan Curriculum

Calculus

30

Ministry of
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Acknowledgements

The Ministry of Education wishes to acknowledge the professional contributions and advice of the provincial curriculum reference committee members:

Bernice Berscheid
Good Spirit School Division
Saskatchewan Teachers' Federation

Bruce Friesen
Living Sky School Division
Saskatchewan Teachers' Federation

Barbara Holzer
Prairie South School Division
Saskatchewan Teachers' Federation

Dasha Kinelovsky
Business and Entrepreneurial Studies Division
SIAST Wascana Campus

Connie Rosowsky
Good Spirit School Division
Saskatchewan Teachers' Federation

Pamela Spock
Regina Public School Division
Saskatchewan Teachers' Federation

Dr. Egan Chernoff
Department of Curriculum Studies
College of Education, University of Saskatchewan

Dr. Edward Doolittle
Associate Professor of Mathematics
First Nations University of Canada

Mark Jensen
North East School Division
Saskatchewan Teachers' Federation

Larry Pavloff
Division Board Trustee, Prairie Spirit School Division
Saskatchewan School Boards Association

Dr. Rick Seaman
Mathematics Education
Faculty of Education, University of Regina

Darrell Zaba
Christ the Teacher Catholic School Division
League of Educational Administrators, Directors, and Superintendents

In addition, the Ministry of Education acknowledges the guidance of the working group members:

Edith Arthur
Prairie Spirit School Division
Saskatchewan Teachers' Federation

Bruce Friesen
Living Sky School Division
Saskatchewan Teachers' Federation

Murray Henry
Prince Albert Roman Catholic School Division
Saskathchewan Teachers' Federation

Lindsay Collins
South East Cornerstone School Division
Saskatchewan Teachers' Federation

Lorne Gottselig
Horizon School Division
Saskatchewan Teachers' Federation

Cameron Milner
Saskatoon Public School Division
Saskatchewan Teachers' Federation

Introduction

Calculus 30 is to be allocated 100 hours. Students must receive the full amount of time allocated to their mathematical learning and the learning should be focused upon them attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade.

The outcomes in the Calculus 30 course are based upon students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

The outcomes in this curriculum define content that is considered a high priority in fields of study and areas of work for which Calculus is appropriate. The outcomes represent the ways of thinking or behaving like a mathematics discipline area expert in those fields of study or areas of work. The mathematical knowledge and skills acquired through this course will be useful to students in many applications throughout their lives in both work and non-work settings.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things that a student needs to understand and/or be able to do to achieve the learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things students should understand or be able to do if they have attained an outcome.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students.

Within the outcomes and indicators in this curriculum, the terms "including" and "such as" as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, consider the indicator, "Develop, explain, and apply strategies for determining characteristics including symmetry, direction, and end behaviour of functions from their equations and from their graphs." It is expected that students will explore the characteristics listed; however, students can also discuss slope, intercepts, or sign analysis to further their understanding of the function. It is not the expectation that all additional characteristics be included within the assessment.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, consider the indicator, "Solve situational problems involving optimization as they apply to topics such as area, volume, and cost." In this case, examples involving area, volume, and cost are possible considerations. It does not exclude students from engaging in other examples and explanations related to optimization, nor is it mandatory that they cover all categories listed.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, consider the indicator, "Develop, explain, and apply strategies (e.g., direct substitution, factoring, simplifying, rationalizing) to determine limits, at real numbers and infinity, of functions including absolute value, root, and piecewise." The examples listed are possible strategies students use to determine limits; however, students can use any available strategy when evaluating limits.

Also included in this curriculum is information pertaining to how the Calculus 30 course connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

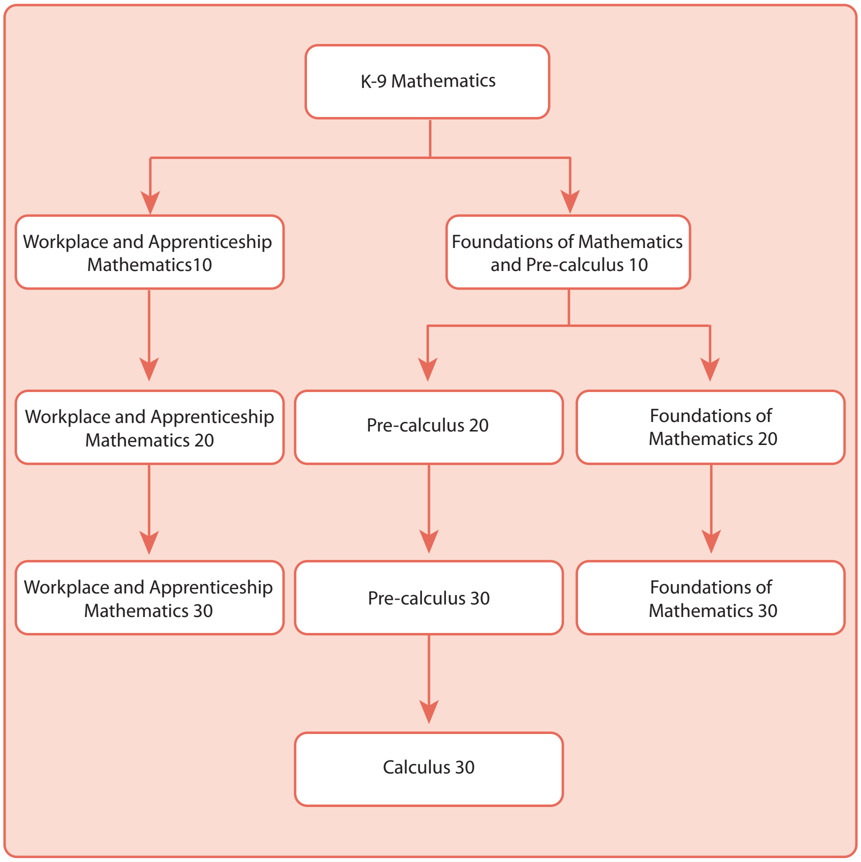
In addition, teachers will find discussions about the critical characteristics of mathematics education, inquiry in mathematics, and assessment and evaluation of student learning in mathematics.

Grades 10-12 Mathematics Framework

Saskatchewan's grades 10 to 12 mathematics curricula are based upon the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10 - 12 Mathematics* (2008). This framework was developed in response to data collected from WNCP post-secondary institutions and business and industry sectors regarding the mathematics needed by students for different disciplines, areas of study, and work areas. From these data, there emerged groupings of areas which required the same types of mathematics. Each grouping also required distinct mathematics so that, even if the same topic was needed in more than one of the groupings, it needed to be addressed in different ways.

The result was the creation of a set of pathways consisting of a single grade 10, 11, and 12 course for each of these groupings which were named Workplace and Apprenticeship Mathematics, Pre-calculus, and Foundations of Mathematics. During the defining of the content for these pathways and courses, it became evident that the content for Grade 10 Foundations of Mathematics and Grade 10 Pre-calculus was very similar. The result is the merging of the two Grade 10 courses (Foundations of Mathematics, and Pre-calculus) into a single course entitled Foundations of Mathematics and Pre-calculus 10. The chart below visually illustrates the courses in each pathway and their relationship to each other.

10-12 Mathematics Pathway Framework



It is important to note that there are no arrows connecting courses in different pathways. This is because the content is different between the pathways, so students wishing to change pathways need to first get the prerequisite courses for the pathway. For example, if students are in or have taken Pre-calculus 20, they cannot move directly into either Foundations of Mathematics 30 or Workplace and Apprenticeship Mathematics 30. In addition, if students have not already taken Workplace and Apprenticeship Mathematics 10, they must do so before entering into Workplace and Apprenticeship Mathematics 20.

Each course in each pathway is to be taught and learned to the same level of rigour. No pathway or course is considered "easy math"; rather, all pathways and courses present "different maths" for different purposes.

Students may take courses from more than one pathway for credit. The current credit requirements for graduation from grade 12 are: one 10 level credit and one 20 level credit in mathematics.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2011) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website for various policy and foundation documents.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve understandings, skills, and attitudes related to these Broad Areas of Learning.

Lifelong Learners

Related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Positive Lifestyle*

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
(NCTM, 2000, p. 20)

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 mathematics). It is crucial that the students discover the mathematical knowledge outlined in the curriculum rather than the teacher covering it.

Sense of Self, Community, and Place

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue and reflection are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to mutual understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Related to the following Goals of Education:

- *Understanding and Relating to Others*
- *Self-Concept Development*
- *Spiritual Development*

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together.

Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students, such as trends in climate change, homelessness, health issues (e.g., hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school , community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Related to the following Goals of Education:

- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.
(NCTM, 2000, p. 4)

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals for Developing Thinking:

- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

Related to CEL of Critical and Creative Thinking.

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This occurs most effectively through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if ...?", "Could we find ...?", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

K-12 Goals for Developing Identity and Interdependence:

- *Understanding, valuing, and caring for oneself (intellectually, emotionally, physically, spiritually)*
- *Understanding, valuing, and caring for others*
- *Understanding and valuing social, economic, and environmental interdependence and sustainability.*

Related to CELs of Personal and Social Development and Technological Literacy.

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accepting responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

Developing Literacies

Through their mathematical learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language are to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives; physical movement; oral, written, visual, and other symbolic forms) when exploring mathematical ideas, solving problems, and communicating understandings.

All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

K-12 Goals for Developing Literacies:

- *Constructing knowledge related to various literacies*
- *Exploring and interpreting the world through various literacies*
- *Expressing understanding and communicating meaning using various literacies.*

Related to CELs of Communication, Numeracy, Technological Literacy, and Independent Learning.

Developing Social Responsibility

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment constructed by the teacher and students support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for contextual validity, and strengthened.

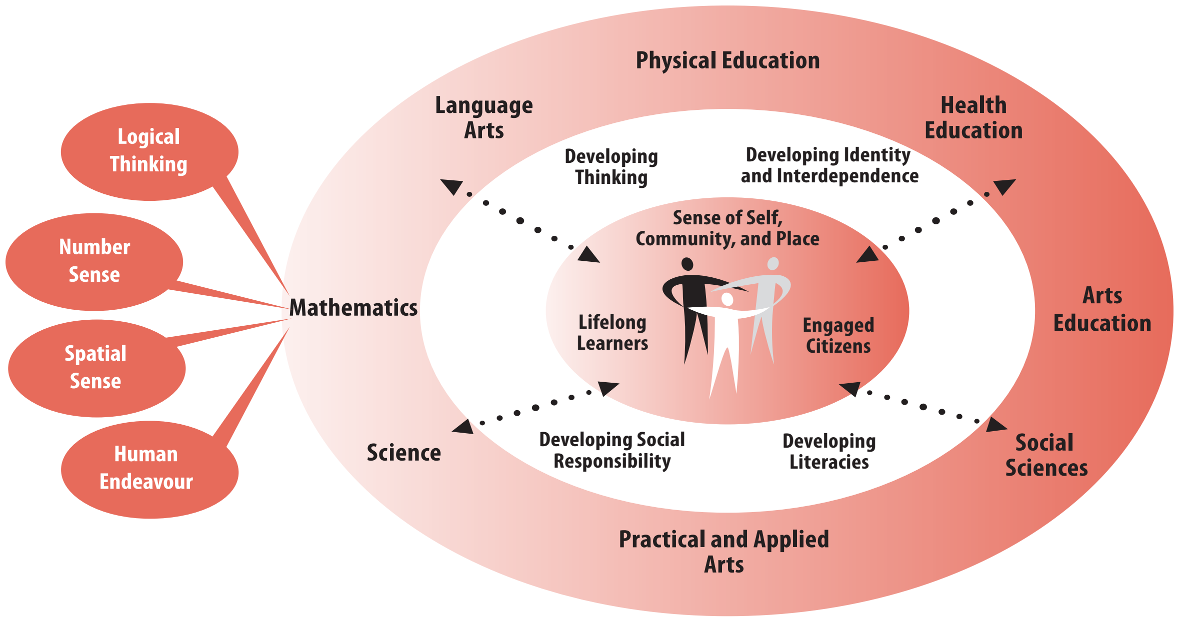
K-12 Goals for Developing Social Responsibility:

- *Using moral reasoning processes*
- *Engaging in communitarian thinking and dialogue*
- *Taking social action.*

Related to CELs of Communication, Critical and Creative Thinking, Personal and Social Development, and Independent Learning.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined above are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes will therefore, also promote student achievement with respect to the K-12 goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and other symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon patterns that are discovered should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Hunman, 1997, p. 10)

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)
- build self-confidence related to mathematical insights and abilities

- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning which values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this curriculum is often viewed in schools and schooling through a Western or European lens, but there are many different lenses, such as those of many First Nations and Métis peoples, through which mathematics can be viewed and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger students will become in their number sense, spatial sense, and logical thinking.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]". This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

Assumptions in this Curriculum

The question in mathematics often arises as to whether students should work with fractions, decimals, or both and, if working with fractions, should mixed numbers or improper fractions be used. For the purposes of this document, we will assume the following:

- If a question or problem is stated with fractions (decimals), the solution should involve fractions (decimals) unless otherwise stated.
- Final fraction solutions can be stated in mixed numbers or improper fractions as long as this is consistent with the original stating of the question or problem.
- The word "or" is used to indicate that students should be able to work with the list of strategies, representations, or approaches given in the list, but they should not be expected to apply more than one of such strategies, representations, or approaches to any given situation or question. For example, in the indicator, "Apply two or more differentiation rules to a function," students could work with more than two differentiation rules in a function; however, it is not expected.

In addition, this curriculum assumes that when engaging in activities related to graphing, the word "sketch" should be used to indicate that the graph can be produced without the use of specific tools or an emphasis on precision. The word "draw" should be used to indicate that specific tools (such as graphing software or graph paper) should be used to produce a graph of greater accuracy.

Critical Characteristics of Mathematics Education

The following sections, in this curriculum, highlight some of the different facets for teachers to consider in the process of changing from "covering" to supporting students in "discovering" mathematical concepts. These facets include:

- the organization of the outcomes
- the seven mathematical processes
- the difference between covering and discovering mathematics
- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Organization of Outcomes

The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: C30.#, where C refers to Calculus, 30 indicates the course level, and where # is the number of the outcome in the list of outcomes. It should be noted, for example, that C30.1 need not be taught before C30.2, nor do the outcomes need to be taught in isolation of each other. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Calculus 30 is at the discretion of the teacher.

Mathematical Processes

This Calculus 30 curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing, along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching. (Caine & Caine, 1991, p.5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase students' willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. When estimating, students need to know what strategy to use, when to use it, and how to use it.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem. (Haylock & Cockburn, 2003, p. 203)

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you ...?", "Can you ...?", or "What if ...?", the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students are given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity. (NCTM, 2000, p. 191)

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Visualization involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.
(Armstrong, 1993, p. 10)

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols and quantities.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies, processes, and rules, as well as the students' current and intuitive understandings of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

For example, when learning in relation to outcome C30.3:

> Demonstrate understanding of limits and continuity.

students can explore, discuss, and analyze authentic situations involving limits. Using a newspaper to determine how many folds are possible before scissors will no longer cut through, determining the minimum number of pieces of duct tape needed to hold a textbook to the wall, or finding the maximum stretch of an elastic band before it snaps are all activities that allow students to discover the meaning of a limit. Students compare different conclusions reached through each activity and whether each goal was attainable and approachable. Once students have discovered the meaning of a limit, the teacher can cover the mathematical conventions used to write limits.

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that are already known or that make sense to students.

Teachers should model appropriate conventional vocabulary. (NCTM, 2000, p. 131)

For example, in outcome C30.4:

> Demonstrate understanding of differentiation based on slope as a rate of change.

a "derivative" and its applications will be new to students. Rather than providing the students with a textbook definition, students construct understanding by developing the meaning through instructional methods such as concept attainment. Students are familiar with the concept of slope as a rate of change and may discuss and analyze a sample of secant lines at a particular point. After students have developed and analyzed the slope of the tangent line, the mathematical terminology of a "derivative" can be introduced and published definitions can be critiqued.

In helping students develop their working mathematical language, it is also important for teachers to recognize that for many students, including First Nations and Métis, they may not recognize a specific term or procedure, but may in fact have a deep understanding of the mathematical topic. Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to formal mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

Teachers must recognize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understanding. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught in relation to their schema, cultural and environmental context, or real life experiences.

A first step in the actualization of mathematics from First Nations and Métis perspectives is empowering teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional Western European ways of teaching mathematics also are culturally biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understanding and mathematical self-confidence and ability through a more holistic and constructivist approach to teaching. Teachers need to pay close attention to those factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

Teachers must recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding, and as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas among cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. In addition, they also allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. As well, ethnomathematics demonstrates the relationship between mathematics and cultural anthropology.

Individually and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthens the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, consider the indicator:

> Critique the statement: "All polynomials can be factored."

Factoring over the set of rational numbers can lead to the answer that not all polynomials can be factored. Extending factoring over the set of real numbers to include irrational numbers and negative exponents allows students to deepen understanding and further develop number sense. Students can explore and discuss the reasons for factoring a polynomial as they justify their answer to the above statement.

Critiquing statements is an effective way to assess students individually, as a small group, or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important, in learning mathematics, that students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, teachers must be prepared to engage students at different points along the continuum.

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area: "What is the surface area of your computer?" Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures), or in both ways.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged. (NCTM, 2000, p. 139)

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you represent what you've done here using mathematical symbols?) or by having students compare their representations with others in the class. In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models.

Making connections also involves looking for patterns. For example, in outcome C30.3:

> Demonstrate understanding of limits and continuity.

Students already have prior knowledge of finite and infinite series from Pre-calculus 20 and can make connections to the patterns previously analysed when discovering infinite geometric sequences. Prior knowledge of patterns allows students to extend their understanding in order to explain the meaning of a limit. Horizontal and vertical asymptotes can also be explored further so students make connections to the concept of continuity and look closely at graphs to develop removable, jump, or infinite discontinuities.

Characteristics of Good Math Homework

- *It is accessible to [students] at many levels.*
- *It is interesting both to [students] and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to, and types of, math homework offered to [students] throughout the year.*

(Adapted from Raphel, 2000, p. 75)

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help to consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of drill will vary among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding in Calculus 30, consider outcome C30.1:

Extend understanding of functions including:

- algebraic functions (polynomial, rational, power)
- transcendental functions (exponential, logarithmic, trigonometric)
- piecewise functions, including absolute value.

Homework that challenges students to match equations and graphs, as well as identifying characteristics such as even, odd, increasing, and decreasing can be assigned and taken up for discussion in class the following day. Small groups of students share how they analyzed the graphs and classifications of each function. When the teacher presents the matches to the students, further engagement into the process of inquiry can occur as students justify the choices made by the teacher. It is then the work of the students to come to consensus on the characteristics of algebraic, transcendental, and piecewise functions.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information to teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Feedback can take many different forms. Instead of saying, "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about ...?"
(Stiff, 2001, p. 70)

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, function notation is something which the teacher will have to show and name for the students; however, first, the students could explore the ideas important for working with function notation.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences .... A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.
(Haylock & Cockburn, 2003, p. 18)

It is important that a mathematics learning environment include effective interplay of:

- reflecting
- exploring of patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

What might you hear or see in a Calculus 30 classroom that would indicate to you that students are developing a deep understanding?

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.
(Mills & Donnelly, 2001, p. xviii)

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

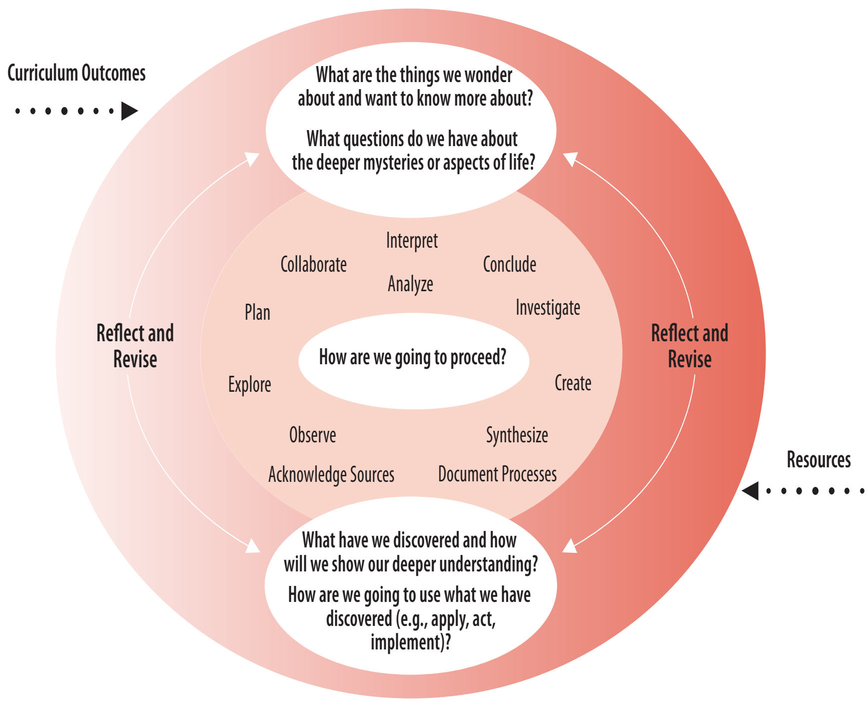
- construct deep knowledge and deep understanding rather than passively receiving it
- are involved and engaged directly in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding

- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answer*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons*
- *spark meaningful connections with prior learning and personal experiences*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry, and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When or how might you know when a function is continuous?"
- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning should also be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.
- are open-ended, whether in answer or approach, as there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but encourage students to make connections and generalizations.

- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p. 3)

Reflection and Documentation of Inquiry

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.
(Schuster & Canavan Anderson, 2005, p. 1)

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, models, works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
C30.1 Extend understanding of functions including: • **algebraic functions (polynomial, rational, power)** • **transcendental functions (exponential, logarithmic, trigonometric)** • **piecewise functions, including absolute value.** [C, CN, ME, R, T, V]	a. Identify functions as algebraic, transcendental, and piecewise from their graphs and from their equations. b. Identify functions as being even, odd, increasing, decreasing, one-to-one, and many-to-one from their graphs and from their equations. c. Critique the statement, "The graph of an absolute value function will be located entirely above the x-axis." d. Develop, generalize, explain, and apply strategies for determining the domain of a function from its equation and from its graphical representation. e. Develop, generalize, explain, and apply strategies for determining the range of a function from its equation and from its graphical representation. f. Identify and express the domain and range of a function using set and interval notation. g. Develop, explain, and apply strategies for determining characteristics including symmetry, direction, and end behaviour of functions from their equations and from their graphs. h. Analyze rational functions to determine conditions where x intercepts, vertical asymptotes, and holes exist by identifying the values of the domain that produce values which are zero, undefined, or indeterminate. i. Critique the statement, "If the denominator of a rational function equals zero at $x = a$, then the rational function has a vertical asymptote at $x = a$."

Goals: Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
C30.2 Extend understanding of factoring, absolute value, and solving inequalities to include: • **rational expressions** • **double inequalities** • **absolute value inequalities.** [C, CN, ME, R]	a. Extend and apply factoring by greatest common factor (GCF) to include negative and rational exponents. b. Extend and apply factoring over the set of rational numbers to the set of real numbers. c. Critique the statement, "All polynomials can be factored."

C30.2 continued

d. Develop, explain, and apply strategies for factoring sum of cubes, difference of cubes, and x^n-y^n (where n is a positive integer).

e. Develop, explain, and apply strategies for solving inequalities containing rational expressions.

f. Develop, explain, and apply strategies for solving double inequalities.

g. Develop, explain, and apply strategies for solving rational equations which include absolute value expressions.

h. Develop, explain, and apply strategies for solving absolute value inequalities containing rational expressions.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes

C30.3 Demonstrate understanding of limits and continuity.

[C, CN, ME, PS, R, T, V]

Indicators

a. Develop and explain the meaning of a limit.

b. Explain the difference between the limit of a function and the value of a function.

c. Critique the statement, "If a function has a limit of L as x approaches a, then as x-values get close to a, the y-values of the function will get progressively closer to L."

d. Determine the value of a limit and express the value using limit notation when given:
   - a graph
   - an algebraic expression.

e. Analyze the graph of a function to determine if it is continuous.

f. Analyze the equation of a function to determine if it continuous.

g. Develop, explain, and apply strategies for determining if a function is continuous at a given point.

h. Develop, explain, and apply strategies for determining the type of discontinuity (removable, jump, or infinite) when given:
   - a graph
   - an algebraic expression.

i. Develop, explain, and apply strategies (e.g., direct substitution, factoring, simplifying, rationalizing) to determine limits, at real numbers and infinity, of functions including absolute value, root, and piecewise.

j. Identify the conditions under which a limit does not exist.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
C30.4 Demonstrate understanding of differentiation based on slope as a rate of change. **[C, CN, PS, R, V]**	a. Identify and explain situations in which slope is used to describe a rate of change. b. Interpret and explain the difference between average rate of change and instantaneous rate of change. c. Solve situational problems involving average rates of change and instantaneous rates of change. d. Develop, explain, and apply strategies for determining the slope of the tangent line at a particular point by finding the slopes of secant lines. e. Develop, explain, and apply the following definition of a derivative: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. f. Develop, explain, and apply rules of differentiation: • power • product • quotient • chain. g. Apply two or more differentiation rules to a function. h. Critique the statement, "It is possible to differentiate any function with the rules that we have studied." i. Identify the value(s) of x where a function is not differentiable. j. Critique the statement, "If a function is continuous, then it is differentiable." k. Develop, explain, and apply the process of implicit differentiation. l. Determine the equation of the tangent line and normal line at a specific point on a function. m. Express derivatives using a variety of notations such as f′(x), y′, d/dx, and dy/dx. n. Critique the statement, "The f′(x) notation for the derivative is superior to the dy/dx notation."

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes

C30.5 Extend understanding of curve sketching by applying differentiation and limits.

[C, CN, V]

Indicators

a. Develop, explain, and apply strategies for finding higher order derivatives and their notations.

b. Develop, explain, and apply strategies for using the first derivative to determine:
   - critical points
   - increasing and decreasing intervals.

c. Develop, explain, and apply strategies for using the second derivative to determine:
   - points of inflection
   - concavity intervals.

d. Describe the difference between relative and absolute extrema.

e. Apply the first and second derivatives to determine relative and absolute extrema.

f. Analyze graphical representations of f(x) to identify critical point(s), increasing and decreasing intervals, point(s) of inflection, and concavity intervals.

g. Identify characteristics of f(x) when given the graph of the first derivative and/or second derivative.

h. Identify characteristics of f(x) when given descriptions of the first derivative and/or second derivative.

i. Apply understanding of limits to determine vertical and horizontal asymptotes.

j. Sketch the graph of a function with and without the use of technology.

k. Critique the statement, "An absolute maximum or minimum value occurs at x=a, if and only if $f'(a) = 0$."

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
C30.6 Demonstrate understanding of the application of derivatives to solve problems including: • **optimization** • **rates of change** • **related rates.** **[C, CN, ME, PS, V]**	a. Solve situational problems involving optimization as they apply to topics such as area, volume, and cost. b. Solve situational problems involving rates of change as they apply to topics such as business, motion, and science. c. Solve situational problems involving related rates by the application of implicit differentiation. d. Critique the statement, "If the rate of change of the radius of a circle doubles, then the rate of change of the area will also double."

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
C30.7 Demonstrate understanding of transcendental function derivatives and their applications. **[C, CN, R, T, V]**	a. Develop an understanding of *e* using limits. b. Utilize the squeeze theorem to evaluate limits involving trigonometric functions. c. Develop, explain, and apply strategies to determine derivatives of the following transcendental functions: • exponential and logarithmic functions using any base • sine and cosine functions. d. Apply first and second derivatives to sketch graphs of: • exponential and logarithmic functions using any base • sine and cosine functions • composite transcendental functions. e. Critique the statement, "A function and its derivative are always different." f. Solve situational problems involving the derivatives of transcendental functions.

Goals: Logical Thinking, Number Sense, Spatial Sense, Mathematics as a Human Endeavour

Outcomes

C30.8 Demonstrate understanding of indefinite and definite integration:

- **by sight**
- **by substitution**
- **as used in the Fundamental Theorem of Calculus.**

[C, CN, ME, PS, T, V]

Indicators

a. Distinguish between indefinite and definite integration.

b. Critique the statement, "If a function can be differentiated, then it can be integrated."

c. Determine indefinite integrals by sight.

d. Determine indefinite integrals by substitution.

e. Apply the Fundamental Theorem of Calculus to evaluate definite integrals by sight and by substitution.

f. Solve situational questions involving integration.

g. Critique the statement, "The integral of f′(x)dx equals f(x)."

h. Develop, explain, and apply strategies for determining the area bounded by:

- a curve and the x-axis over [a,b]
- two curves.

i. Critique the statement, "To integrate any power, we apply in reverse the power rule for differentiation."

Assembling evidence from a variety of sources is more likely to yield an accurate picture.
(NCTM, 2000, p. 24)

Assessment should not merely be done to students; rather it should be done for students.
(NCTM, 2000, p. 22)

What are examples of assessments as learning that could be used in Calculus 30 and what would be the purpose of those assessments?

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- Achievement of provincial curriculum outcomes
- Effectiveness of teaching strategies employed
- Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Calculus 30. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the content of Calculus 30.

Assessment should become a routine part of the ongoing classroom activity rather than an interruption. (NCTM, 2000, p. 23)

Glossary

Double Inequalities: A double inequality consists of three expressions separated from each other by two inequalities (e.g., $3 < x + 6 < 4x$).

Infinite Discontinuity: The function f(x) has an infinite discontinuity at x-values that yield a/0 for f(x) where $a \neq 0$.

Jump Discontinuity: The function f(x) has a jump discontinuity if the limits taken from the left and right sides are finite but not equal.

Removable Discontinuity: The function f(x) has a removable discontinuity at x = b if f(x) can be defined or redefined at x = b so that it can be made continuous.

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Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Calculus 30 Curriculum

1. Please indicate your role in the learning community:

☐ parent ☐ teacher ☐ resource teacher

☐ guidance counsellor ☐ school administrator ☐ school board trustee

☐ teacher librarian ☐ school community council member

☐ other ____

What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

☐ print

☐ online

b) Please indicate which format(s) of the curriculum you prefer:

☐ print

☐ online

3. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
appropriate for its intended purpose	1	2	3	4
suitable for your use	1	2	3	4
clear and well organized	1	2	3	4
visually appealing	1	2	3	4
informative	1	2	3	4

4. Explain which aspects you found to be:

Most useful:

Least useful:

5. Additional comments:

6. Optional:

   Name: ____

   School: ____

   Phone: _____________________ Fax: _____________________

   E-mail: ____

Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

Executive Director
Student Achievement and Supports Branch/
Réussite et soutien des élèves
Ministry of Education
2220 College Avenue
Regina SK S4P 4V9
Fax: 306-787-2223